Exponent notation

We use exponent notation to show repeated multiplication by the same number.

For example:

we can use exponent notation to write **2 × 2 × 2 × 2 × 2** as

2^5 (base: 2; Exponent or power: 5)

This number is read as 'two to the power of five'.

$2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$

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Multiplying numbers in exponent form

When we multiply two numbers written in exponent form and with the same base we can see an interesting result.

For example:

$3^4 \times 3^2 = (3 \times 3 \times 3 \times 3) \times (3 \times 3)$

$= 3 \times 3 \times 3 \times 3 \times 3 \times 3$

$= 3^6 = 3^{(4+2)}$

$7^3 \times 7^5 = (7 \times 7 \times 7) \times (7 \times 7 \times 7 \times 7 \times 7)$

$= 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7$

$= 7^8 = 7^{(3+5)}$

When we **multiply** two numbers with the **same base** the exponents are **added**. In general, $x^m \times x^n = x^{(m+n)}$

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Dividing numbers in exponent form

When we divide two numbers written in exponent form and with the same base we can see another interesting result.

For example:

$4^5 \div 4^2 = \frac{\cancel{4} \times \cancel{4} \times 4 \times 4 \times 4}{\cancel{4} \times \cancel{4}} = 4 \times 4 \times 4 = 4^3 = 4^{(5-2)}$

$5^6 \div 5^4 = \frac{\cancel{5} \times \cancel{5} \times \cancel{5} \times \cancel{5} \times 5 \times 5}{\cancel{5} \times \cancel{5} \times \cancel{5} \times \cancel{5}} = 5 \times 5 = 5^2 = 5^{(6-4)}$

When we **divide** two numbers with the **same base** the exponents are **subtracted**. In general, $x^m \div x^n = x^{(m-n)}$

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Raising a power to a power

Sometimes numbers can be raised to a power and the result raised to another power.

For example,

$(4^3)^2 = 4^3 \times 4^3$

$= (4 \times 4 \times 4) \times (4 \times 4 \times 4)$

$= 4^6 = 4^{(3 \times 2)}$

When a number is raised to a power and then raised to another power, the powers are multiplied. In general, $(x^m)^n = x^{mn}$

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The power of 0

Look at the following division:

$6^4 \div 6^4 = 1$

Using the second exponent law,

$6^4 \div 6^4 = 6^{(4-4)} = 6^0$

That means that: $6^0 = 1$

Any non-zero number raised to the power of 0 is equal to 1.

For example,

$10^0 = 1$ $3.452^0 = 1$ $723\,538\,592^0 = 1$

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Exponent laws

Here is a summary of the exponent laws you have met so far:

$x^m \times x^n = x^{(m+n)}$

$x^m \div x^n = x^{(m-n)}$

$(x^m)^n = x^{mn}$

$x^1 = x$

$x^0 = 1$ (for $x \neq 0$)

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Negative exponents

Look at the following division:

$$3^2 \div 3^4 = \frac{\cancel{3} \times \cancel{3}}{\cancel{3} \times \cancel{3} \times 3 \times 3} = \frac{1}{3 \times 3} = \frac{1}{3^2}$$

Using the second exponent law,

$$3^2 \div 3^4 = 3^{(2-4)} = 3^{-2}$$

That means that $3^{-2} = \frac{1}{3^2}$

Similarly, $6^{-1} = \frac{1}{6}$ $7^{-4} = \frac{1}{7^4}$ and $5^{-3} = \frac{1}{5^3}$

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Exponent laws for negative exponents

Here is a summary of the exponent laws for negative exponents.

$x^{-1} = \frac{1}{x}$ → The reciprocal of x is $\frac{1}{x}$

$x^{-n} = \frac{1}{x^n}$ → The reciprocal of x^n is $\frac{1}{x^n}$

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Fractional exponents

Exponents can also be fractional. Suppose we have $9^{\frac{1}{2}}$.

In general, $x^{\frac{1}{2}} = \sqrt{x}$

In general, $x^{\frac{1}{3}} = \sqrt[3]{x}$

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Fractional indices

What is the value of $25^{\frac{3}{2}}$?

We can think of $25^{\frac{3}{2}}$ as $25^{\frac{1}{2} \times 3}$.

Using the rule that $(x^a)^b = x^{ab}$ we can write

$$25^{\frac{1}{2} \times 3} = (\sqrt{25})^3$$
$$= (5)^3$$
$$= 125$$

In general,

$$x^{\frac{m}{n}} = (\sqrt[n]{x})^m$$

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Exponent laws for fractional exponents

Here is a summary of the exponent laws for fractional exponents.

$x^{\frac{1}{2}} = \sqrt{x}$

$x^{\frac{1}{n}} = \sqrt[n]{x}$

$x^{\frac{m}{n}} = \sqrt[n]{x^m}$ or $(\sqrt[n]{x})^m$

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